Electron Transport in Four-Terminal Cross Junction with Potential Barrier in Magnetic Fields

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Abstract

We study the electron transport and the Hall resistance in a four-terminal cross junction in magnetic fields. We consider the case in which the potential barrier exists at the junction so that electrons can be transmitted to other leads only by tunneling. The calculated Hall resistance shows strange behavior which is completely different from the standard Hall bar measurement. At lower magnetic field, the Hall resistance increases with the magnetic field. As the field increases, after taking the peak value, the Hall resistance decreases even to the negative values. Further increase of field causes the repeated change of the sign of the Hall resistance and it shows the oscillatory-like behavior. We discuss this anomalous behavior of the Hall resistance in term of the transmission probabilities from one lead to other leads. The probability density and the current density in the potential are also examined.

1. Introduction

Recent advances in microfabrication technique have realized many systems which show quantum effects. Quantum wires fabricated in the two-dimensional electron gas (2DEG) in the semiconductor heterostructures are one of those systems in which the electron transport is strongly effected by quantum mechanics and in the ballistic transport it can be treated as the waveguide of electron waves [1]. Some experiments on the Hall resistance in quantum wires have shown anomalous behavior like the quenching of the Hall resistance [2] and the geometrical influence of the Hall bar to the Hall resistance [3]. Theoretical analyses have been done [4-7] and revealed that they were notpurely classical but they could be related to the classical orbit of electrons in magnetic fields.

In this paper, we consider a purely quantum-mechanical situation, that is, the electron transport through tunneling barrier in magnetic fields. In quantum wires propagating waves are localized at the edge (edge state) and this can be understood in term of the Lorentz force. However, in tunneling region, it is not obvious to which an electron turns or if it turns or not. We are going to analyze the behavior of electrons in tunneling and the Hall resistance measured in such systems theoretically. The model of the calculation is given in Sec. II. We show results and discuss about them in Sec. III. The brief summary is given in the last section.

2. Model

We consider the electron transport in the four-terminal cross junction in 2DEG (Fig. 1).Similar systems have been analyzed

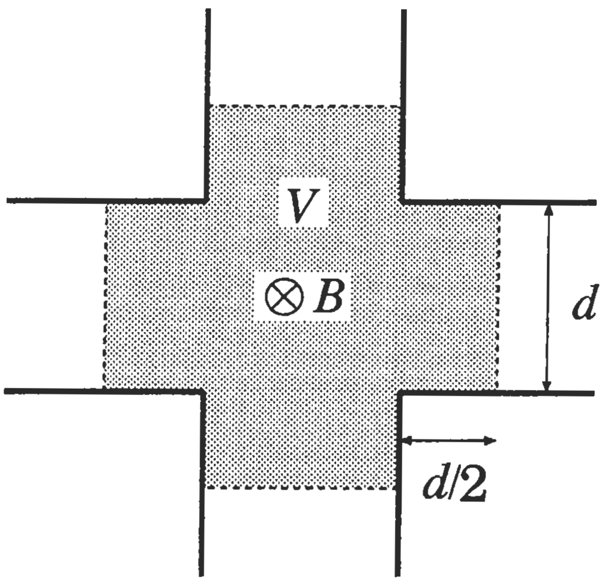

Fig.1 Schematic of a four-terminal Hall bar.The finite potential V exists at the hatched region.The walls of leads are infinite potential.

experimentally and theoretically [1]. The difference from those theoretical analyses is caused by the positive potential, V, at the cross so that the kinetic energy of electrons becomes smaller or even negative corresponding to the value of the potential. A uniform magnetic field is applied perpendicular to the 2DEG,$\mathbf{B}$=(0,0,-B). The leads are assumed to be formed by the infinite potential and the width of the lead is d. We have solved the Schrödinger equation for an electron in this potential. For the numerical calculation we used a method called the boundary element method [7]. The boundary condition is that the incident wave comes from one of the leads and scattered to four leads, and the wave function vanishes at the walls. From the solved wave function the transmission probabilities from the incident lead to other leads are obtained. The Hall resistance can be calculated with the the Landauer-Büttiker formula [8];

$$R_H = \frac{h}{e^2} \frac{T_R - T_L}{(T_R+T_F)^2+(T_L+T_F)^2} \qquad (1)$$

where T_R(T_F, T_L) is the transmission probability to the lead on the right (forward, left) of the incident lead. In this paper we assume that the magnetic field is piercing from the face to the back of Fig. 1 and the Lorentz force makes the moving electron turn to right. This is the reason of the difference of the numerator in Eq. 1 is $T_R - T_L$ (If the magnetic field is piercing from the back to the face that part becomes $T_L - T_R$).

3. Results and Discussions

The wave number, $k \equiv \sqrt{2mE}/\hbar$, of an incident electron is $1.8\pi/d$. Int[kd/π] is the number of propagating subbands at the straight waveguide with width d in the absence of a magnetic field. There is only one propagating subband at the straight part in this case. We use the wave number value rather than energy itself hereafter, and the value of the potential is also given by the dimension of wave number, $k_u \equiv \sqrt{2mV}/\hbar$.

We calculated two values of the potential at the center; $k_u d/\pi = 1.7$ and 1.9. It is needless to say that $k_u = 1.9\pi/d$ is completely the tunneling region. When $k_u = 1.7\pi/d$ it is still smaller than the energy of incident electron but the transmission through the junction is effectively tunneling. In this value the wave number, k_p, which corresponds to the kinetic energy at the potential is $\sqrt{k^2 - k_u^2} = 1.86/d$ and is less than π/d. This means no propagation subband at the potential.

In Fig. 2, we plot the transmission probabilities in linear scale (a), in log scale (b), and the Hall resistance (c) for the case of $k_u = 1.7\pi/d$. Same plots for $k_u = 1.9\pi/d$ are in Fig. 3.

The behavior of transmission probabilities are very similar in Figs. 2 and 3, though it differs about two orders of magnitude. Transmission

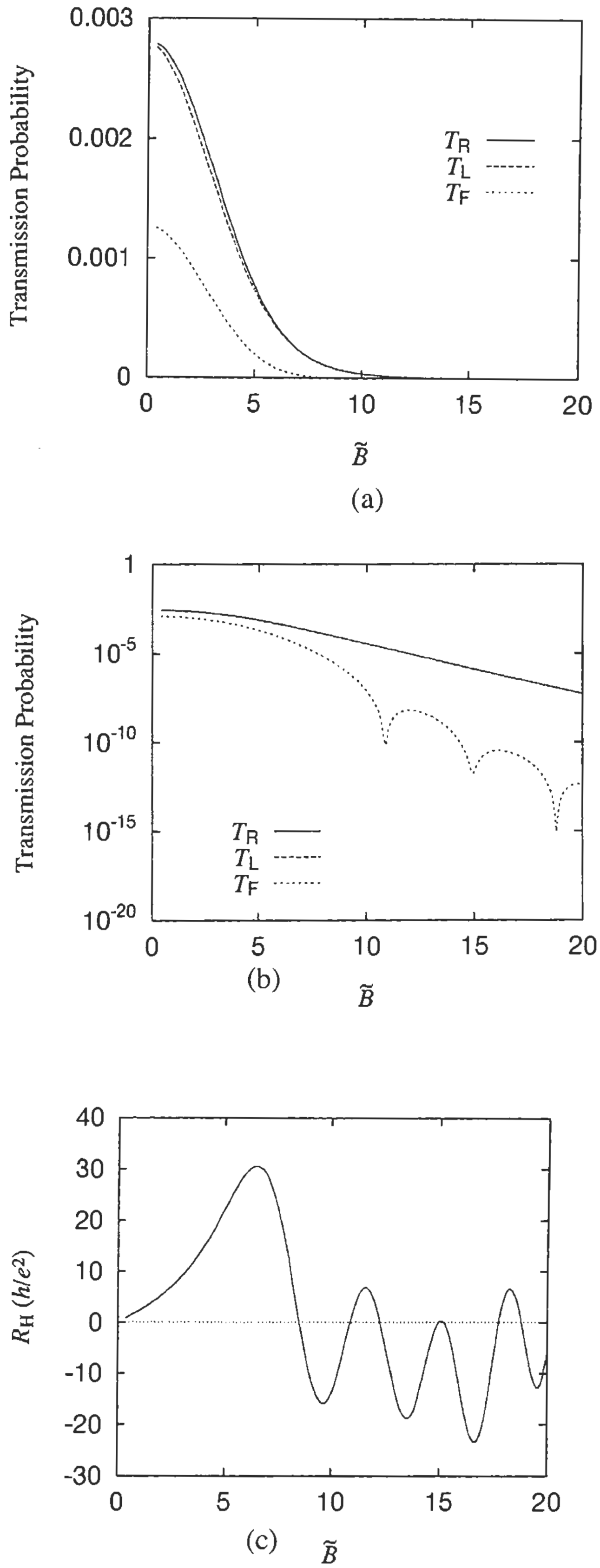

Fig.2 Transmission probabilities in linear scale (a) and in log scale (b) and Hall resistance (c) as a function of $\tilde{B}$. $k=1.8\pi/d$ and $k_u=1.7\pi/d$.

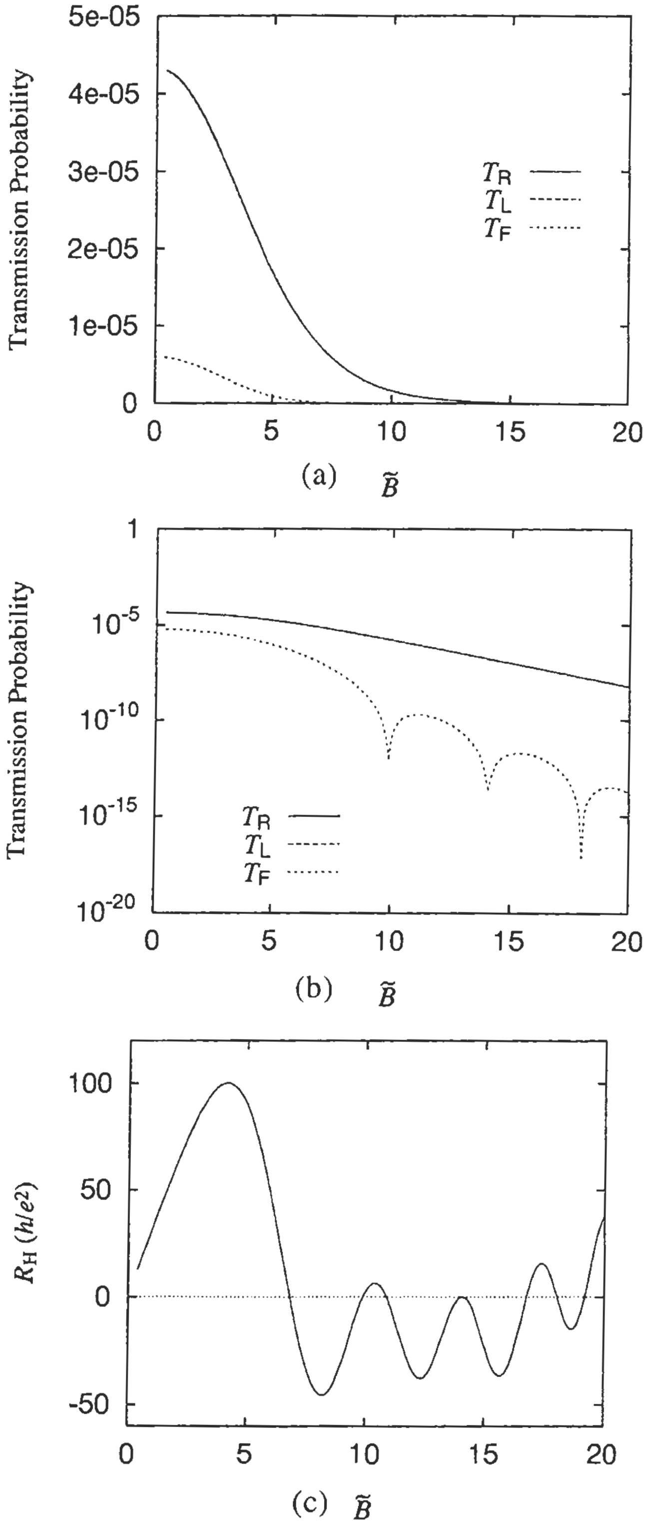

Fig.3 Transmission probabilities in linear scale (a) and in log scale (b) and Hall resistance (c) as a function of $\tilde{B}$. k=1.8 π /d and $k_{//}$=1.9 π /d.

probabilities decrease rapidly with the magnetic field. The value of T_F becomes much smaller than other two, T_R and T_L. This can be understood since it is the tunneling transmission and the forward lead is further from the incident lead than other two.There are some

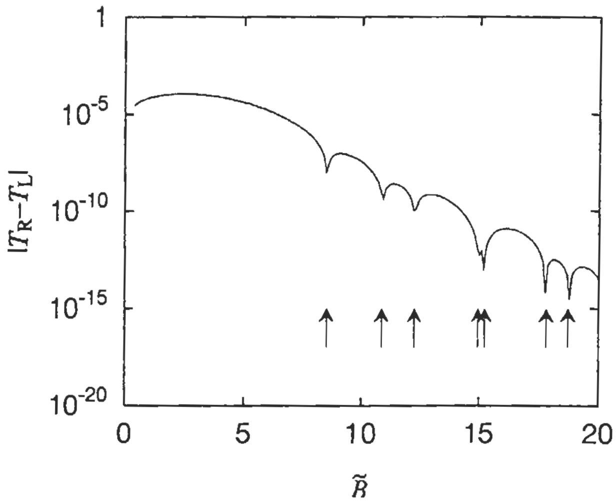

Fig.4 The absolute difference of transmission probabilities $|T_R - T_L|$ as a function of $\tilde{B}$. The arrows indicate the values at which the sign of the Hall resistance changes.

sharp dips in the plot of T_F in the log scale where it almost vanishes.

The Hall resistance is also very similar to each other except the magnitude. It increases with the magnetic field in low field but after taking the maximum value it decreases even to negative values. After taking a minimum it increases again and the Hall resistance repeats changing its sign. As a result, the Hall resistance shows oscillatory-like behavior in high field region. From Eq. 1 the change of sign of the Hall resistance means the change of the sign of the numerator of the equation, T_R - T_L. In the Hall bar without the potential at the center T_R usually becomes larger than T_L due to the Lorentz force and the Hall resistance is positive. The experiment by Ford and co-workers on quantum wires showed the negative Hall resistance [3]. It is caused by the geometrical effect, called the rebound effect [3,5,7], and is related to classical orbits of electrons. The phenomenon found in this calculation may be different from that negative Hall resistance by the rebound effect since this is in the tunneling region and classical orbits have no means here. It is interesting that some points where the sign of the Hall resistance changes Figs. 2(c) and 3(c) coincide with the dip of the T_F plot in Figs. 2(b) and 3(b), respectively. In other words, whenever the dip of T_F appears the Hall resistance changes its sign. These behavior of transmission probabilities and the Hall resistance may be the characteristics of the electron transport through tunneling barrier in the magnetic field.

In Fig. 4 we plot $|T_R - T_L|$ in log scale for $k_{\parallel}=1.7\pi/d$. We indicated by arrows the values when $T_R - T_L$ (and hence the Hall resistance) changes its sign. From this plot the difference of them becomes smaller as the field increases. However, as in Fig. 2(c), the magnitude of Hall resistance is not so changed. It is because of the smallness of the transmission probabilities. From Eq. 1, besides physical constants, it is the difference of transmission probabilities divided by the square of the transmission probabilities. Though $|T_R - T_L|$ becomes smaller the denominator is also smaller and as a result the order of the Hall resistance does not change so much as the transmission probabilities. We plot the probability density at two magnetic field values in Fig. 5 by contours. One is at $\tilde{B}=2.5$(a) where $T_R - T_L$ takes the maximum value and the other is at $\tilde{B}=9.1$(b) where $T_R - T_L$ takes the maximum negative value. The incident wave comes from the left lead. Though the Hall resistance at these values are completely different (positive and negative), we cannot find any difference between these contour plots except the decaying constants. The difference of T_R and T_L is so small that the contours look almost symmetric. We plot the current of the probability density in the tunneling region at these two values of field in

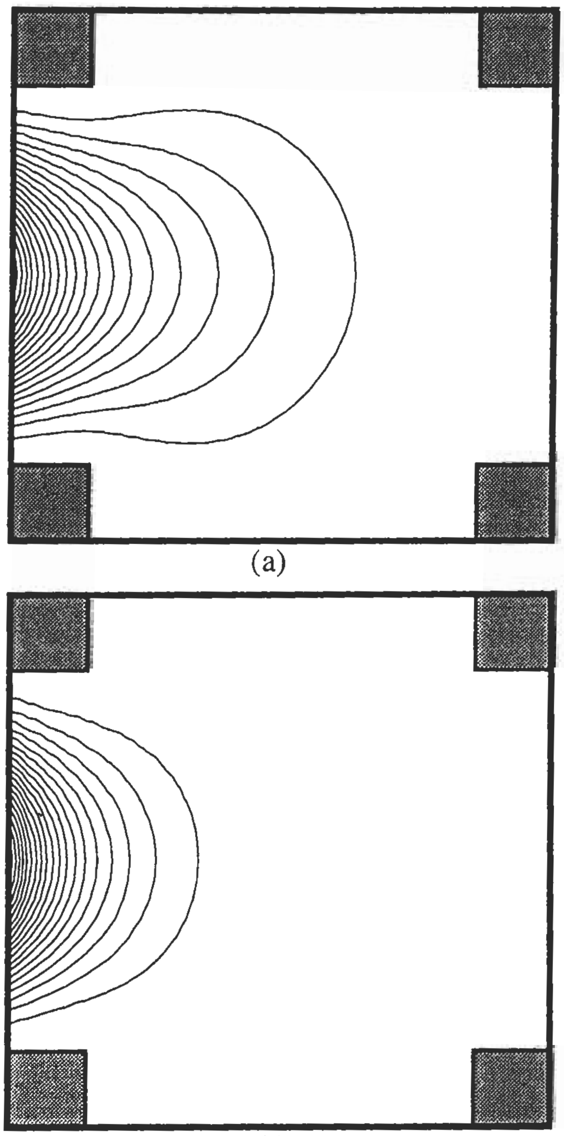

Fig.5 The contours of the probability density of an electron when $(T_R - T_L)$ takes the maximum values and negative maximum in Fig.4. $k_{\parallel}=1.7\pi/d$ and (a) $\tilde{B}=2.5$ (b) $\tilde{B}=9.1$. The incident wave comes from the left.

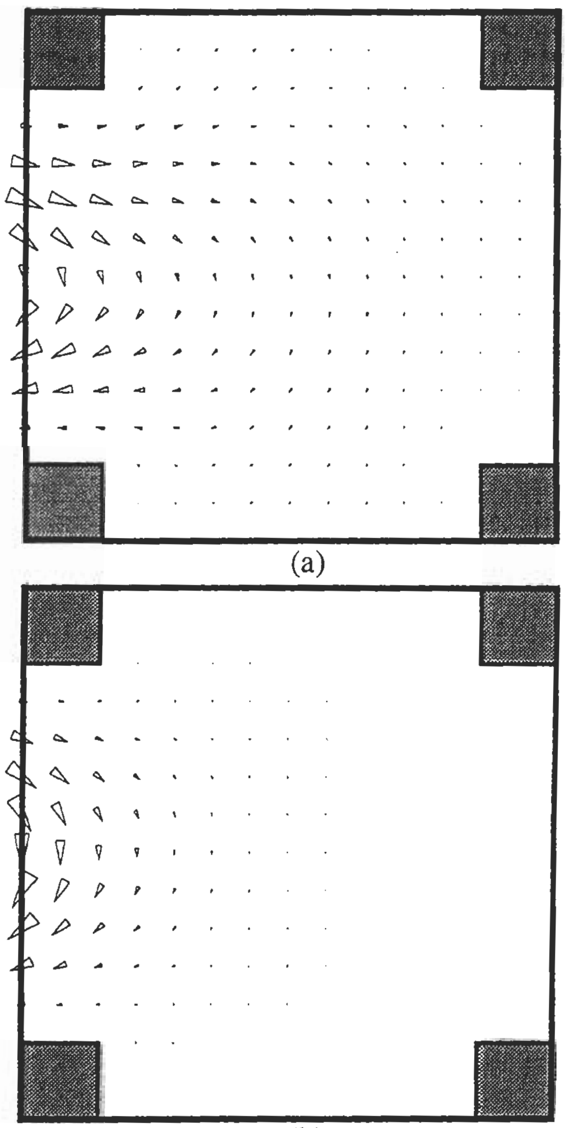

Fig.6 The current density plots of an electron. $k_{\parallel}=1.7\pi/d$ and (a) $\tilde{B}=2.5$ (b) $\tilde{B}=9.1$. The triangular arrows are normalized by the maximum value of current density in each plot.

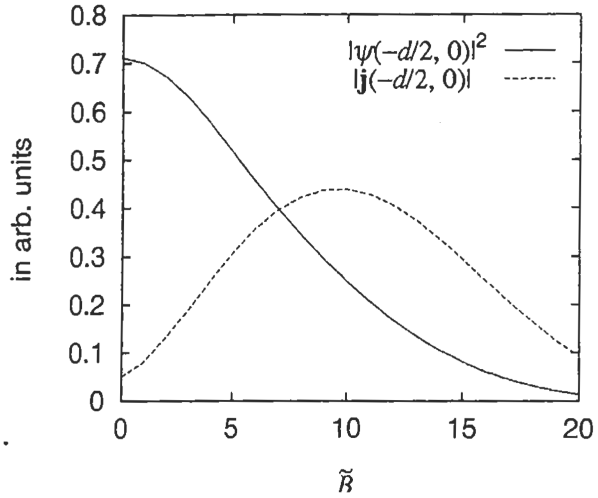

Fig.7 The probability density and current density at $\mathbf{r}=(-\frac{d}{2},0)$ as a function of $\tilde{B}$.

Fig. 6; (a) $\tilde{B}$=2.5, (b) $\tilde{B}$=9.1. The length of triangular arrows in Fig. 6 corresponds to the value of local current density and it is scaled by the maximum value in each plot. An interesting point in this figure is that the current is rotating and back to the incident lead. Furthermore, the value of the local current density becomes larger in (b) than in (a) though the transmission probabilities to other leads are smaller in (b) than in (a). This means large current flows in but reflected to the incident lead and the transmission is small as a total. This is completely different from the tunneling in the absence of a magnetic field, in which the small transmission probabilities means the small local current. We plot the probability density and current density at $\mathbf{r}=(-d/2,0)$ in Fig.7. In this range of magnetic field the probability density becomes smaller as the field increases but the current density is alway larger than that in the absence of a magnetic field.

4. Conclusion

We have studied the electron transport through tunneling barrier in the Hall bar in magnetic fields. It shows many interesting behavior which cannot be observed in the usual Hall bar without the barrier. The Hall resistance changes its sign as the magnetic field value increases and it shows oscillatory-like behavior. It is caused by the change of sign of the difference between the transmission probabilities to the right and that to the left leads. The transmission to the forward lead is much smaller than other two and it shows some sharp dips.These dips are always accompanied by the change of the sign of the Hall resistance. The transmission probabilities at the potential becomes smaller as the magnetic field increases but local current density in the potential barrier becomes larger in the low field region.

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